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バナッハ空間におけるファジィ微分方程式の解に関する初期値連続依存性について

Continuous Dependence in Parameters of Solutions for Fuzzy Differential Equations of a Banach Space

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Abstract

We introduce a parametric representation of fuzzy numbers with bounded supports as well as we consider a Banach space including the set of fuzzy numbers, where the addition in the Banach space is the same one due to the extension principle but the difference and scalar products are not the same as those of the principle. In this article we treat initial value problems of fuzzy differential equations and give existence and uniqueness theorems and sufficient conditions for the continuous dependence with respect to initial conditions of solutions.

1 Introduction

Let $I = [0,1]$. Denote a set of fuzzy numbers with bounded supports by $\mathcal{F}_b^{st}$ as follows (e.g. [6, 7]): The following definition means that a fuzzy number can be identified with a membership function.

Definition 1.1 *Denote a set of fuzzy numbers with bounded supports and strict fuzzy convexity by*

$$\mathcal{F}_b^{st} = \{\mu : \mathbf{R} \to I \text{ satisfying (i)-(iv) below}\}.$$

(i) μ *has a unique number* $m \in \mathbf{R}$ *such that* $\mu(m) = 1$ (*normality*);

(ii) $supp(\mu) = cl(\{\xi \in \mathbf{R} : \mu(\xi) > 0\})$ *is bounded in* $\mathbf{R}$ (*bounded support*);

(iii) μ *is strictly fuzzy convex on* $supp(\mu)$ *as follows:*

(a) *if* $supp(\mu) \neq \{m\}$, *then*
$$\mu(\lambda\xi_1 + (1-\lambda)\xi_2) > \min[\mu(\xi_1), \mu(\xi_2)]$$
for $\xi_1, \xi_2 \in supp(\mu)$ *with* $\xi_1 \neq \xi_2$ *and* $0 < \lambda < 1$;

(b) *if* $supp(\mu) = \{m\}$, *then* $\mu(m) = 1$ *and* $\mu(\xi) = 0$ *for* $\xi \neq m$;

(iv) μ *is upper semi-continuous on* $\mathbf{R}$ (*upper semi-continuity*).

μ is called a membership function if $\mu \in \mathcal{F}_b^{st}$. Fuzzy numbers are identified by membership functions. In what follows we denote the $\alpha-$cut sets of μ by

$$\mu_\alpha = L_\alpha(\mu) = \{\xi \in \mathbf{R} : \mu(\xi) \geq \alpha\}$$

for $\alpha \in (0,1]$. By the extension principle due to Zadeh, the binary operation between fuzzy numbers is nonlinear. It does not necessarily hold that $(k_1+k_2)\mu = k_1\mu + k_2\mu$ for a membership function $\mu \in \mathcal{F}_b^{st}$ and $k_i \in \mathbf{R}, i = 1,2$ with $k_1 + k_2 > 0, k_1 < 0 < k_2$.

We introduce the following parametric representation of $\mu \in \mathcal{F}_b^{st}$ as

$$x_1(\alpha) = \min L_\alpha(\mu), \quad x_2(\alpha) = \max L_\alpha(\mu)$$

for $0 < \alpha \leq 1$ and

$$x_1(0) = \min supp(\mu), \quad x_2(0) = \max supp(\mu).$$

From the strict fuzzy convexity it can be seen that a fuzzy number $x = (x_1, x_2)$ means a bounded continuous curve over $\mathbf{R}^2$ and $x_1(\alpha) \leq x_2(\alpha)$ for $\alpha \in I$ (see [8].)

In Section 2 we show that the set of fuzzy numbers $\mathcal{F}_b^{st}$ construct a Banach space by the Puri-Ralescue's method .

In Section 3 we discuss differentiation and integration of fuzzy functions. In the case of differentiation our representation of fuzzy numbers is enable to calculate addition , scalar product and difference without difficulties, but it is not easy to calculate the difference by the extension principle. Moreover we define the integral of fuzzy functions by calculating end-points of $\alpha-$cut sets.

In Section 4 we treat initial value problems of fuzzy differential equations $x' = f(t,x)$. We give existence and uniqueness theorems of the fuzzy differential equations and we show sufficient conditions for the continuous dependence with respect to initial conditions of solutions.

2 Induced Normed Space of Fuzzy Numbers

Let $g : \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ be an $\mathbf{R}-$valued function. The corresponding binary operation of two fuzzy numbers $x, y \in \mathcal{F}_b^{st}$ to $g(x,y) : \mathcal{F}_b^{st} \times \mathcal{F}_b^{st} \rightarrow \mathcal{F}_b^{st}$ is calculated by the extension principle of Zadeh. The membership function $\mu_{g(x,y)}$ of g is as follows:

$$\mu_{g(x,y)}(\xi) = \sup_{\xi = g(\xi_1, \xi_2)} \min(\mu_1(\xi_1), \mu_2(\xi_2))$$

Here $\xi, \xi_1, \xi_2 \in \mathbf{R}$ and μ_1, μ_2 are membership functions of x, y, respectively. From the extension principle, it follows that, in case where $g(x,y) = x + y$,

$$\begin{aligned}
&\mu_{x+y}(\xi) \\
&= \max_{\xi=\xi_1+\xi_2} \min_{i=1,2} (\mu_i(\xi_i)) \\
&= \max\{\alpha \in I : \xi = \xi_1 + \xi_2,\ \xi_i \in L_\alpha(\mu_i), i = 1,2\} \\
&= \max\{\alpha \in I : \xi \in [x_1(\alpha) + y_1(\alpha), x_2(\alpha) + y_2(\alpha)]\}.
\end{aligned}$$

Thus we get

$$x + y = (x_1 + y_1, x_2 + y_2).$$

In the similar way we have

$$x - y = (x_1 - y_2, x_2 - y_1).$$

Denote a metric by

$$d(x,y) = \sup_{\alpha \in I} \max(|x_1(\alpha) - y_1(\alpha)|, |x_2(\alpha) - y_2(\alpha)|)$$

for $x = (x_1, x_2), y = (y_1, y_2) \in \mathcal{F}_b^{st}$.

Theorem 2.1 *$\mathcal{F}_b^{st}$ is a complete metric space in $C(I)^2$.*

Proof See [8].

According to the extension principle of Zadeh, for respective membership functions μ_x, μ_y of $x, y \in \mathcal{F}_b^{st}$ and $\lambda \in \mathbf{R}$, the following addition and a scalar product are given as follows :

$$\begin{aligned}
\mu_{x+y}(\xi) &= \sup\{\alpha \in [0,1] : \\
&\quad \xi = \xi_1 + \xi_2,\ \xi_1 \in L_\alpha(\mu_x), \xi_2 \in L_\alpha(\mu_y)\}; \\
\mu_{\lambda x}(\xi) &= \begin{cases} \mu_x(\xi/\lambda) & (\lambda \neq 0) \\ 0 & (\lambda = 0,\ \xi \neq 0) \\ \sup_{\eta \in \mathbf{R}} \mu_x(\eta) & (\lambda = 0,\ \xi = 0) \end{cases}
\end{aligned}$$

In [5] they introduced the following equivalence relation $(x,y) \sim (u,v)$ for $(x,y),(u,v) \in \mathcal{F}_b^{st} \times \mathcal{F}_b^{st}$, *i.e.*,

$$(x,y) \sim (u,v) \Longleftrightarrow x+v = u+y. \tag{2.1}$$

Putting $x = (x_1, x_2), y = (y_1, y_2), u = (u_1, u_2), v = (v_1, v_2)$ by the parametric representation, the relation (2.1) means that the following equations hold.

$$x_i + v_i = u_i + y_i \quad (i = 1, 2)$$

Denote an equivalence class by $\langle x,y \rangle = \{(u,v) \in \mathcal{F}_b^{st} \times \mathcal{F}_b^{st} : (u,v) \sim (x,y)\}$ for $x, y \in \mathcal{F}_b^{st}$ and the set of equivalence classes by

$$(\mathcal{F}_b^{st})^2/\sim = \{\langle x,y \rangle : x, y \in \mathcal{F}_b^{st}\}$$

such that one of the following cases (i) and (ii) hold:

(i) if $(x,y) \sim (u,v)$, then $\langle x,y \rangle = \langle u,v \rangle$;

(ii) if $(x,y) \not\sim (u,v)$, then $\langle x,y \rangle \cap \langle u,v \rangle = \emptyset$.

Then $(\mathcal{F}_b^{st})^2/\sim$ is a linear space with the following addition and scalar product

$$\langle x,y \rangle + \langle u,v \rangle = \langle x+u, y+v \rangle \tag{2.2}$$

$$\lambda\langle x,y \rangle = \begin{cases} \langle \lambda x, \lambda y \rangle & (\lambda \geq 0) \\ \langle (-\lambda)y, (-\lambda)x \rangle & (\lambda < 0) \end{cases} \tag{2.3}$$

for $\lambda \in \mathbf{R}$ and $\langle x,y \rangle, \langle u,v \rangle \in (\mathcal{F}_b^{st})^2/\sim$. They denote a norm in $(\mathcal{F}_b^{st})^2/\sim$ by

$$\| \langle x,y \rangle \| = \sup_{\alpha \in I} d_H(L_\alpha(\mu_x), L_\alpha(\mu_y)).$$

Here d_H is the Hausdorff metric is as follows:

$$d_H(L_\alpha(\mu_x), L_\alpha(\mu_y)) = \max(\sup_{\xi \in L_\alpha(\mu_x)} \inf_{\eta \in L_\alpha(\mu_y)} |\xi - \eta|, \sup_{\eta \in L_\alpha(\mu_x)} \inf_{\xi \in L_\alpha(\mu_y)} |\xi - \eta|)$$

It can be easily seen that $\| \langle x,y \rangle \| = d(x,y)$. Note that $\| \langle x,y \rangle \| = 0$ in $(\mathcal{F}_b^{st})^2/\sim$ if and only if $x = y$ in $\mathcal{F}_b^{st}$.

3 Fuzzy Differential and Fuzzy Integral

In this section we consider fuzzy function in a Banach space induced by the normed space $(\mathcal{F}_b^{st})^2/\sim$. It can be seen that for $x, y \in \mathcal{F}_b^{st}$

$$\langle x,y \rangle = \langle x,0 \rangle + \langle 0,y \rangle = \langle x,0 \rangle - \langle y,0 \rangle.$$

Denoting a set of fuzzy numbers by

$$X_0 = \{\langle x,0 \rangle \in (\mathcal{F}_b^{st})^2/\sim : x, 0 \in \mathcal{F}_b^{st}\},$$

which is a Banach space (see *e.g.*, [8]). Then we have $(\mathcal{F}_b^{st})^2/\sim = X_0 - X_0$.

Denote the completion of $(\mathcal{F}_b^{st})^2/\sim$ by X. Let J be an interval in $\mathbf{R}$. In what follows we consider a function $f : J \to X$ as $f = \langle (f_1, f_2), 0 \rangle$. Here f has the parametric representation of $f = (f_1, f_2)$, where $f_i(t, \alpha)$ for $i = 1, 2$ are the end-points of the $\alpha-$cut set of f In this section we give definitions of differentiation and integration of fuzzy functions.

A fuzzy function $f : J \to X$ is said to be differentiable at $t_0 \in J$, if there exists an $\eta \in X$ such that for any $\varepsilon > 0$ there exists a $\delta > 0$ satisfying

$$\| \frac{f(t) - f(t_0)}{t - t_0} - \eta \| < \varepsilon$$

for $t \in J$ and $0 < |t - t_0| < \delta$. Denote $\eta = f'(t_0) = \frac{df}{dt}(t_0)$. f is differentiable on J if f is differentiable at any $t \in J$. In the similar way higher order derivatives of f are defined by $f^{(k)} = (f^{(k-1)})'$ for $k = 2, 3, \cdots$. (Cf. [2, 3])

In [5] they define the embedding $j : \mathcal{F}_b^{st} \to X$ such that $j(u) = \langle u, 0 \rangle$. The function $f : J \to \mathcal{F}_b^{st}$ is called differentiable in the sense of Puri-Ralescu, if $j(f(\cdot))$ is differentiable. Suppose that f is differentiable at $t \in J$ in the above sense, denoted the differential $f'(t) \in \mathcal{F}_b^{st}$. Then we have $\frac{d}{dt}(j(f(t))) = \langle f'(t), 0 \rangle$, *i.e.*, f is differentiable in the sense of Puri-Ralescu. In [4, 5] H-difference and H-differentiation of f is treated as follows. Suppose that for $f(t+h), f(t) \in \mathcal{F}_b^{st}$, there exists $g \in \mathcal{F}_b^{st}$ such that $f(t+h) = f(t) + g$, then g is called to the H-difference, denoted $f(t+h) - f(t)$. The function f is called H-differentiable at $t \in J$ if there exists an $\eta \in \mathcal{F}_b^{st}$ such that both $\lim_{h \to +0} \frac{f(t+h) - f(t)}{h}$ and $\lim_{h \to +0} \frac{f(t) - f(t-h)}{h}$ exist and equal to η. If f is H-differentiable, then $f'(t) = \eta$.

Proposition 3.1 *If f is differentiable at t_0, then f is continuous at t_0.*

Theorem 3.1 *Denote a parametric representation of f by $f = \langle (f_1, f_2), 0 \rangle$. Here f_1, f_2 are functions defined on $I \times J$ to $\mathbf{R}$ and the left-, right-end point of the α-cut set $L_\alpha(f(t))$. If f is differentiable at t_0, then it follows that there exist $\frac{\partial}{\partial t} f_1(t, \alpha)$, $\frac{\partial}{\partial t} f_2(t, \alpha)$ and that*

$$f'(t_0) = (\frac{\partial}{\partial t} f_1, \frac{\partial}{\partial t} f_2)(t_0).$$

Theorem 3.2 *It follows that $f'(t) \equiv 0$ if and only if $f(t) \equiv const \in X$.*

In the following definition we give one of integrals of fuzzy functions.

Definition 3.1 *Let $J = [a, b]$ and f be a mapping from J to X. Divide the interval J such that $a = t_0 < t_1 < \cdots < t_n = b$ and $\tau_i \in [t_{i-1}, t_i]$ for $i = 1, 2, \cdots, n$. f is integrable over J if there exists the limit $\lim_{|\Delta| \to 0} \sum_{i=1}^{n} f(\tau_i)\Delta_i$, where $\Delta_i = t_i - t_{i-1}, |\Delta| = \max_{1 \le i \le n} \Delta_i$. Define*

$$\int_a^b f(s)ds = \lim_{|\Delta| \to 0} \sum_{i=1}^{n} f(\tau_i)\Delta_i.$$

Proposition 3.2 *Let f be integrable over J. Then the following statements* (i)-(ii) *hold.*

(i) *f is bounded on J, i.e., there exists an $M > 0$ such that $\| f(t) \| \le M$ for $t \in J$.*

(ii) *If $f(t) \in X$ for $t \in J$, then $\int_a^t f(s)ds \in X$ for $t \in J$.*

Proposition 3.3 *If f is continuous on $[a, b]$ then f is integrable over the interval.*

Theorem 3.3 *Let $f : J \to X$ with $f = \langle (f_1, f_2), 0 \rangle$ be integrable over $[a, b]$. Then it follows that*

$$\int_a^b f(s)ds = \langle (\int_a^b f_1(s)ds, \int_a^b f_2(s)ds), 0 \rangle$$

Conversely, if f_1, f_2 are continuous on $[a, b] \times I$, then f is integrable over $[a, b]$.

Proposition 3.4 *Let f be continuous on the interval $[a, b]$.*
Denote $F(t) = \int_a^t f(s)ds$. Then the following properties (i) *and* (ii) *hold.*

(i) *F is differentiable on $[a, b]$ with $F(t) \in X$ and $F' = f$;*

(ii) *For $t_1, t_2 \in [a, b]$ and $t_1 \le t_2$, we have $\int_{t_1}^{t_2} f(s)ds = F(t_2) - F(t_1)$.*

Proposition 3.5 *Let f is continuous on $[a,b]$. Then it follows that*

$$\| \int_a^b f(s)ds \| \leq \int_a^b \| f(s) \| ds.$$

Theorem 3.4 *Let $f : [a,b] \to X$ be continuous on $[a,b]$ and differentiable on (a,b), Then it follows that there exists a number $c \in (a,b)$ such that*

$$\| f(b) - f(a) \| \leq (b-a) \| f'(c) \| .$$

Definition 3.2 *Let $f : J \to X^n$ such that $f(t) = (f_1(t), f_2(t), \cdots, f_n(t))^T$. f is differentiable on J if each f_i is differentiable on J for $i = 1, 2, \cdots, n$. Define the derivative $f'(t) = (f_1'(t), f_2'(t), \cdots, f_n'(t))^T$.*

Let $f : [a,b] \to X^n$ such that
$f(t) = (f_1(t), f_2(t), \cdots, f_n(t))^T$. f is integrable over $[a,b]$ if f_i is integrable over $[a,b]$ for $i = 1, 2, \cdots, n$. Define the integral

$$\int_a^b f(s)ds = (\int_a^b f_1(s)ds, \int_a^b f_2(s)ds, \cdots, \int_a^b f_n(s)ds)^T.$$

It can be easily proved that similar theorems and propositions concerning to X^n−valued functions to ones in this section hold.

4 Fuzzy Differential Equations

In this section we consider the initial value problems of the following type of fuzzy differential equation

$$x'(t) = f(t, x(t)) \tag{4.4}$$
$$x(t_0) = x_0. \tag{4.5}$$

Here $f : \mathbf{R} \times X^n \to X^n, t_0 \in \mathbf{R}, x_0 \in X^n$.

We denote the initial value problem of higher order fuzzy differential equations by

$$\begin{aligned} x^{(n)} &= f(t, x(t), x'(t), \cdots, x^{(n-1)}(t)) \\ x^{(k)}(t_0) &= x_k, \quad k = 0, 1, \cdots, n-1, \end{aligned} \tag{4.6}$$

where $f : \mathbf{R} \times X^n \to X, t_0 \in \mathbf{R}, x_k \in X$. Define $x_1(t) = x(t), x_2(t) = x'(t), \cdots, x_n(t) = x^{(n-1)}(t)$ so that the above problem can be reduced to Problem ((4.4),(4.5)). In this section we show some kinds of conditions to solutions of ((4.4),(4.5)) for the existence, uniqueness and continuation.

Definition 4.1 *Define a norm $\| p \| = \max(|t|, \| x \|)$ for $p = (t,x) \in \mathbf{R} \times X^n$. Let $p_0 \in \mathbf{R} \times X$. Denote a neighborhood of p_0 by $U(p_0, \delta) = \{p \in \mathbf{R} \times X^n : \| p - p_0 \| < \delta\}$ and a relative neighborhood of p_0 by $V(p_0, \delta) = U(p_0, \delta) \cap (\mathbf{R} \times X^n)$ for $\delta > 0$. Let $V \subset \mathbf{R} \times X^n$. V is said to be a relatively open subset in $\mathbf{R} \times X^n$, if for any $p \in V$ there exists a relative neighborhood $V(p) \in \mathbf{R} \times X^n$ such that $V(p) \subset V$. In the similar way we define relatively open subsets in $X^n, X^n \times \mathbf{R}, \mathbf{R} \times X^n \times \mathbf{R}$.*

Consider a function $f : V \to X^n$, where V is a relatively open subset in $\mathbf{R} \times X^n$. f is said to satisfy a locally Lipschitz condition if for any $p = (t_0, x_0) \in V$ there exists a relative neighborhood $V(p) \subset V$ and a number $L_p > 0$ such that

$$\| f(t, x_1) - f(t, x_2) \| \leq L_p \| x_1 - x_2 \|$$

for $(t, x_1), (t, x_2) \in V(p)$.

Theorem 4.1 *Let $f : V \to X^n$ satisfy the locally Lipschitz condition and be continuous on V. Then there exists one and only one solution x of ((4.4),(4.5)) defined on $[t_0, t_0 + r]$ passing through $p = (t_0, x_0) \in V$, where $r > 0$.*

Suppose that the same conditions of Theorem 4.1 hold. Denote an interval $\mathcal{J} = \{[t_0, T) \in \mathbf{R}$: there exits a solution x of ((4.4),(4.5)) on $[t_0, T)\}$. For $J \in \mathcal{J}$ there exists a unique solution of ((4.4),(4.5)) on J. Denote $J(t_0, x_0) = \cup_{J \in \mathcal{J}} J$ and $x_f(t_0, x_0, t) = x_J(t)$ for $t \in J \in \mathcal{J}$. For $t \in J(t_0, x_0)$ there exists a unique value $x_J(t)$. The function $x_f : V \times J(t_0, x_0) \to X^n$ is said to be the solution of ((4.4),(4.5)) with the maximal interval $J(t_0, x_0)$. Denote a mapping $x_f : \mathbf{R} \times X^n \times \mathbf{R} \to X^n$ defined on $D(f) = \{(t_0, x_0, t) : (t_0, x_0) \in V, t \in J(t_0, x_0)\}$. See [9].

Theorem 4.2 *Suppose that the same conditions of Theorem* 4.1 *hold. Let* $J = [t_0, T] \subset J(t_0, x_0) \cap J(t_0, x_0^{'})$, *where* $T > t_0$. *Then there exists an* $M > 0$ *such that*

$$\| x_f(t_0, x_0^{'}, t) - x_f(t_0, x_0, t) \| \leq M \| x_0^{'} - x_0 \|$$

for $t \in J$.

Consider the following fuzzy differential equation

$$x^{'}(t) = f(x(t)). \tag{4.7}$$

Corollary 4.1 *Let* $f : V \rightarrow X^n$ *satisfy the locally Lipschitz condition on* V, *where* $V \in X^n$ *is a relatively open subset. Then there exists one and only one solution* x *of* $((4.7), (4.5))$ *defined on* $[t_0, t_0 + r]$ *passing through* $p = (t_0, x_0) \in V$, *where* $r > 0$.

In the similar discussion concerning (4.4) the maximal interval $J(t_0.x_0)$ and the corresponding to solution x_f can be defined for $(t_0, x_0) \in \mathbf{R} \times V$ (see [9]). It can be seen that

$$\begin{aligned} J(t_0, x_0) &= J(0, x_0) + t_0 \\ &= \{t + t_0 : t \in J(0, x_0)\} \end{aligned}$$

and for $t \in J(t_0, x_0)$ we get

$$x_f(t_0, x_0, t) = x_f(0, x_0, t - t_0).$$

Thus we denote $J(x_0) = J(0, x_0), x_f(x_0, \cdot) = x_f(0, x_0, \cdot)$ and $D_0(f) = \{(t_0, x_0) \in V \times J(x_0)\}$.

Theorem 4.3 *The same conditions of Corollary* 4.1 *hold. Then* $D_0^+(f) = \{(x_0, t) \in D_0(f) : t > 0\}$ *is a relatively open subset in* $X^n \times \mathbf{R}$ *and the mapping* x_f *is continuous on* $D_0(f)$.

In what follows some type of Lipschiz condition plays an important role in discussing properties of solutions for ((4.4),(4.5)).

Condition (L) *For any* $p = (t_0, x_0) \in V$ *there exists a relative neighborhood* $V(p) \subset V$ *and a number* $L_p > 0$ *such that*

$$\| f(t_1, x_1) - f(t_2, x_2) \| \leq L_p \| (t_1, x_1) - (t_2, x_2) \|$$

for $(t_1, x_1), (t_2, x_2) \in V(p)$.

A function $y : J \rightarrow \mathbf{R} \times X^n$ is said to be differentiable at $t \in J$ if

$$y(t + h) = y(t) + \zeta h + o(h)$$

as $h \rightarrow 0$, where $\zeta \in \mathbf{R} \times X^n$ and $o(h)/h \rightarrow 0$, denoted $\zeta = y^{'}(t)$.

Theorem 4.4 *Consider Problem* ((4.4),(4.5)). *Let* $f : V \rightarrow X^n$ *satisfy Condition* (L), *where* V *is a relatively open subset in* $\mathbf{R} \times X^n$. *Then* $D^+(f) = \{(t_0, x_0, t) \in D(f) : t > t_0\}$ *is a relatively open subset in* $\mathbf{R} \times X^n \times \mathbf{R}$ *and the mapping* x_f *is continuous on* $D(f)$.

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